## EXERCISE – III SUBJECTIVE QUESTIONS

**1.** Evalaute

**(i)** $\int_{-\infty}^{\infty} \frac{dx}{x^2+2x+2}$

**(ii)** $\int_{\sqrt{2}}^{\infty} \frac{dx}{x\sqrt{x^2-1}}$

**(iii)** $\int_0^4 \frac{x^2}{1+x}\,dx$

**2.** Let f(x) = $\ell n\left(\frac{1-\sin x}{1+\sin x}\right)$, then show that

$$\int_a^b f(x)\,dx = \int_b^a \ell n\left(\frac{1+\sin x}{1-\sin x}\right)dx$$

**3.** Evaluate

**(i)** $\int_0^2 [x^2]\,dx$

**(ii)** $\int_{-1}^1 [\cos^{-1}x]\,dx$

**4.** Evalaute

**(i)** $\int_{-1}^1 e^{|x|}\,dx$

**(ii)** $\int_{-\pi/4}^{\pi/4} |\sin x|\,dx$

**(iii)** $\int_{-5}^5 |x+2|\,dx$

**(iv)** $\int_{-\pi/4}^{\pi/4} \frac{x+\pi/4}{2-\cos 2x}\,dx$

**5.** Evaluate

**(i)** $\int_0^1 \sin^{-1}\left(\frac{2x}{1+x^2}\right)dx$

**(ii)** $\int_0^1 \frac{x\tan^{-1}x}{(1+x^2)^{3/2}}\,dx$

**(iii)** $\int_0^1 x^2\sin^{-1}x\,dx$

**(iv)** $\int_0^{\sqrt{3}} \tan^{-1}\left(\frac{2x}{1-x^2}\right)dx$

**6.** Evaluate

**(i)** $\int_0^{\pi/2} \frac{\sin 2\theta\,d\theta}{\sin^4\theta+\cos^4\theta}$

**(ii)** $\int_0^{\pi/2} \sqrt{\cos\theta}\,\sin^3\theta\,d\theta$

**(iii)** $\int_0^{\pi/4} \frac{\sin x+\cos x}{9+16\sin 2x}\,dx$

**7.** Evaluate

**(i)** $\int_a^b \frac{dx}{\sqrt{(x-a)(b-x)}}$

**(ii)** $\int_a^b \sqrt{(x-a)(b-x)}\,dx$

**8.** Evaluate

**(i)** $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx$

**(ii)** $\displaystyle\int_0^{\pi/2} \frac{e^{\sin x}}{e^{\sin x} + e^{\cos x}}\,dx$

**(iii)** $\displaystyle\int_0^{a} \frac{\sqrt{x}}{\sqrt{x} + \sqrt{a-x}}\,dx$

**(iv)** $\displaystyle\int_0^{\pi/2} \frac{a\sin x + b\cos x}{\sin x + \cos x}\,dx$

**9.** Evaluate

**(i)** $\displaystyle\int_{-1}^{2} \{2x\}\,dx$ (where $\{*\}$ denotes fractional part function)

**(ii)** $\displaystyle\int_0^{10\pi} (|\sin x| + |\cos x|)\,dx$

**10.** If f(x) is an odd function defined on $\left[-\frac{T}{2}, \frac{T}{2}\right]$ and has period T, then prove that $\phi(x) = \displaystyle\int_0^{x} f(t)\,dt$ is also periodic with period T.

**11.** If $f(x) = 5^{g(x)}$ and $g(x) = \displaystyle\int_2^{x^2} \frac{t}{\ell n(1+t^2)}\,dt$ then find the value of $f'(\sqrt{2})$

**12.** If $f(x) = \displaystyle\int_0^{\sin^2 x} \sin^{-1}\sqrt{t}\,dt + \int_0^{\cos^2 x} \cos^{-1}\sqrt{t}\,dt$ then prove that $f'(x) = 0\ \forall\ x \in R$.

**13.** Prove that following inequalities

**(i)** $\displaystyle\frac{\sqrt{3}}{8} < \int_{\pi/4}^{\pi/3} \frac{\sin x}{x}\,dx < \frac{\sqrt{2}}{6}$

**(ii)** $\displaystyle 4 \le \int_1^{3} \sqrt{(3+x^3)}\,dx \le 2\sqrt{30}$

**14.** Evaluate

**(i)** $\displaystyle\lim_{n\to\infty} \sum_{r=1}^{n-1} \frac{1}{\sqrt{n^2 - r^2}}$

**(ii)** $\displaystyle\lim_{n\to\infty} \frac{3}{n}\left[1 + \sqrt{\frac{n}{n+3}} + \sqrt{\frac{n}{n+6}} + \sqrt{\frac{n}{n+9}} + \ldots + \sqrt{\frac{n}{n+3(n-1)}}\right]$

**15.** $\displaystyle\int_0^{\pi} e^{\cos^2 x} \cos^3 (2n+1)\ x\,dx,\ n \in I$

**16.** If f, g, h be continuous function on [0, a] such that f(a – x) = f(x), g(a – x) = – g(x) and 3 h(x) – 4h (a – x) = 5, then prove that, $\displaystyle\int_0^{a} f(x)\,g(x)\,h(x) = 0.$

**17.** Show that $\displaystyle\int_0^{x} e^{zx}.e^{-z^2} dz = e^{x^2/4} \int e^{-z^2/4} dz$ .

**18.** Let $f(x) = \begin{cases} 1-x & \text{if } 0 \le x \le 1 \\ 0 & \text{if } 1 < x \le 2 \\ (2-x)^2 & \text{if } 2 < x \le 3 \end{cases}$. Define the function $F(x) = \displaystyle\int_0^{x} f(t)\,dt$ and show that F is continuous in [0, 3] and differentiable in (0, 3).

**19.** Evaluate, $\displaystyle\int_0^{1} |x - t|.\cos \pi t\,dt$ where 'x' is any real number

**20.** Evaluate, $I = \displaystyle\int_0^{1} 2\sin(pt)\ \sin(qt)\,dt$, if :

**(i)** p & q are different roots of the equation, tan x = x.
**(ii)** p & q are equal and either is root of the equation tan x = x.

**21.** If $f(x) = \dfrac{\sin x}{x}\ \forall\ x \in (0, \pi]$, prove that, $\displaystyle\frac{\pi}{2}\int_0^{\pi/2} f(x)\,f\left(\frac{\pi}{2} - x\right) dx = \int_0^{\pi} f(x)\,dx$

**22.** Evaluate $\displaystyle\int_0^{1} \frac{1}{(5 + 2x - 2x^2)(1 + e^{(2-4x)})}\,dx$

**23.** If n > 1, evaluate $\int_0^{\infty} \frac{dx}{(x+\sqrt{1+x^2})^n}$

**24.** $\int_0^1 (\{2x\}-1)(\{3x\}-1)\,dx$ ,

where { $*$ } denotes fractional part of x.

**25.** Prove that $\int_0^x \frac{\sin x}{x+1}\,dx \geq 0$ for $x \geq 0$.

**26.** Let f(x) be a continuous function $\forall\ x \in R$, except at x = 0 such that $\int_0^a f(x)\,dx$ , $a \in R^+$ exists. If $g(x) = \int_x^a \frac{f(t)}{t}\,dt$, prove that $\int_0^a g(x)\,dx = \int_0^a f(x)\,dx$ .

**27.** $\int_0^{\pi} \frac{x\,dx}{9\cos^2 x + \sin^2 x}$

**28.** $\int_0^{\pi/2} \sqrt{\frac{1-\sin 2x}{1+\sin 2x}}\,dx$

**29.** Evaluate $I_n = \int_1^e (\ell n^n\ x)\,dx$ hence find $I_3$.

**30.** $\int_0^{\pi/2} \sin 2x.\text{arc}\tan(\sin x)\,dx$

**31.** $\int_0^{\pi/4} \frac{x\,dx}{\cos x(\cos x + \sin x)}$

**32.** $\int_1^2 \frac{(x^2-1)\,dx}{x^3.\sqrt{2x^4-2x^2+1}} = \frac{u}{v}$ where u and v are in their lowest form. Find the value of $\frac{(1000)u}{v}$.

**33.** Find the value of the definite integral $\int_0^{\pi} |\sqrt{2}\sin x + 2\cos x|\,dx$.

**34.** Evaluate the integral $\int_3^5 \left(\sqrt{x+2\sqrt{2x-4}} + \sqrt{x-2\sqrt{2x-4}}\right)dx$

**35.** If $P = \int_0^{\infty} \frac{x^2}{1+x^4}\,dx$ ; $Q = \int_0^{\infty} \frac{x\,dx}{1+x^4}$ and $R = \int_0^{\infty} \frac{dx}{1+x^4}$ then prove that

**(a)** $Q = \frac{\pi}{4}$

**(b)** P = R

**(c)** $P - \sqrt{2}\,Q + R = \frac{\pi}{2\sqrt{2}}$

**36.** $\int_0^1 \frac{x^4(1-x)^4}{1+x^2}\,dx$

**37.** $\int_0^1 \frac{x^2.\ln x}{\sqrt{1-x^2}}\,dx$

**38.** $\int_{-2}^{2} \frac{x^2-x}{\sqrt{x^2+4}}\,dx$

**39.** $\int_0^{\sqrt{3}} \sin^{-1}\frac{2x}{1+x^2}\,dx$

**40.** $\int_0^{\pi/2} \frac{a\sin x + b\cos x}{\sin\left(\frac{\pi}{4}+x\right)}\,dx$

**41.** $\int_0^{2\pi} \frac{dx}{2+\sin 2x}$